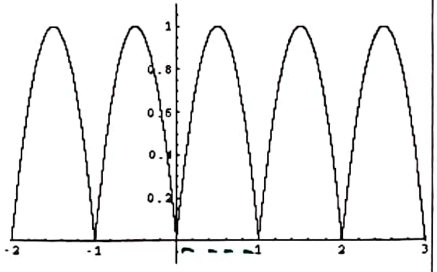
4.4 Graphs of Sine and Cosine Functions

If the values of a function are the same for each given interval of the domain, the function is said to be **periodic**. The interval is the **period** of the function. (smallest interval of x that contains one copy of the repeating pattern)

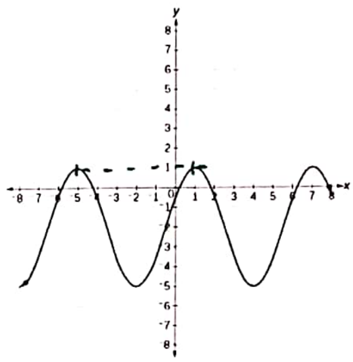
Ex 1: Determine if each function is periodic. If so, state the period.

a)



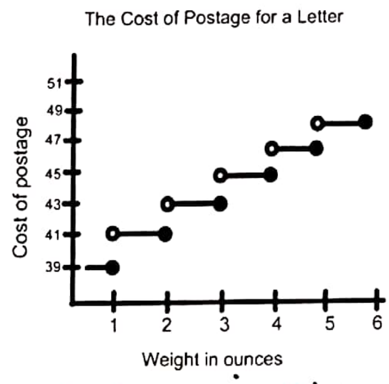
Period = 1

b)



Period: 6

c)

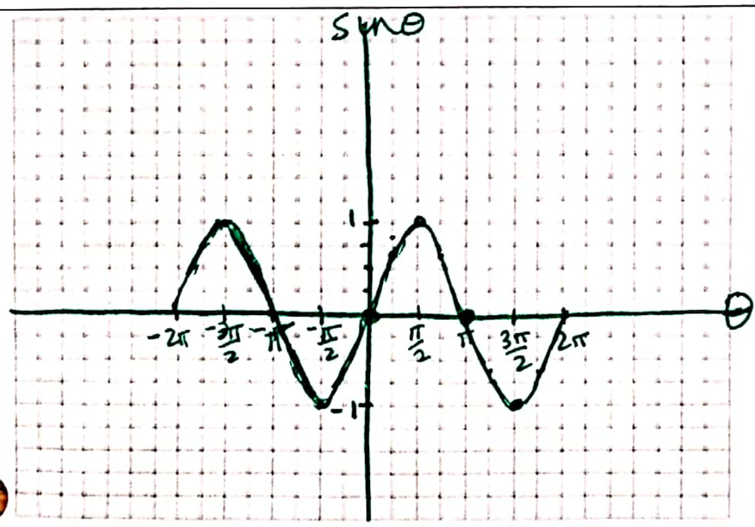


not periodic

Ex 2: Graph the functions $y = \sin\theta$ and $y = \cos\theta$ from -2π to 2π in multiples of $\frac{\pi}{4}$.

θ	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin\theta$ = y	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0

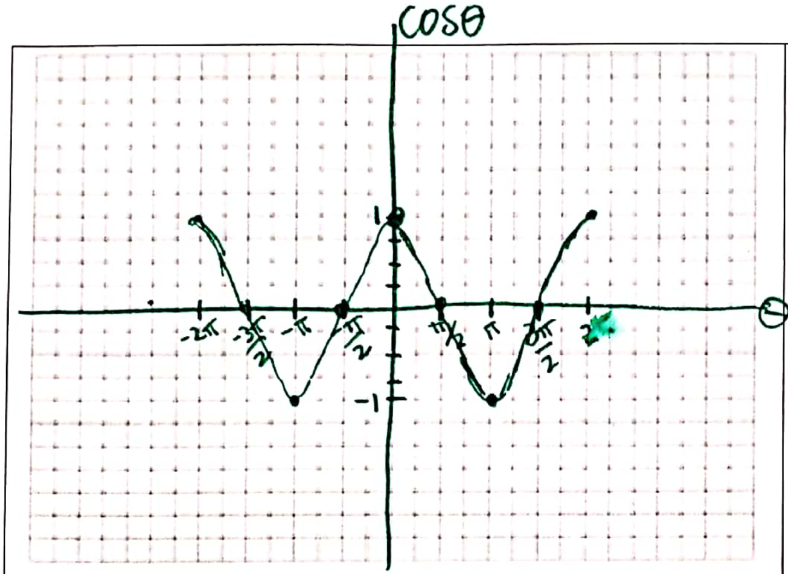
0.707



Properties of the Graph y = sin θ

1. The period is 2π.
2. The domain is the set of real numbers.
3. The range is the set of real numbers between -1 and 1, inclusive.
4. The x-intercepts are located at π n, where n is an integer.
5. The y-intercept is 0.
6. The maximum values are y = 1 and occur when x = $\frac{\pi}{2} + 2\pi$n, where n is an integer.
7. The minimum values are y = -1 and occur when x = $\frac{3\pi}{2} + 2\pi$n, where n is an integer.

θ	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
cos θ x	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1

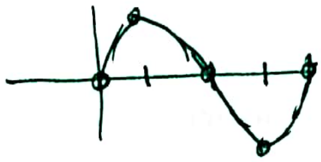


Properties of the Graph y = cos θ

1. The period is 2π.
2. The domain is the set of real numbers.
3. The range is the set of real numbers between -1 and 1, inclusive.
4. The x-intercepts are located at $\frac{\pi}{2}+\pi n$, where n is an integer.
5. The y-intercept is 1.
6. The maximum values are y = 1 and occur when x = π n, where n is an even integer.
7. The minimum values are y = -1 and occur when x = π n, where n is an odd integer.

sinθ

I - M - I - m - I



cosθ

M - I - m - I - M

-sinθ

I - m - I - M - I

-cos θ

m - I - M - I - m

You will be studying the graphic effect of each of the constants *a, b, c,* and *d* in the equations of the forms:

$$y = d + a\sin(bx - c) \qquad \text{and} \qquad y = d + a\cos(bx - c).$$

The constant factor *a* in $y = a\sin x$ acts as a scaling factor – a vertical stretch or vertical shrink of the basic sine curve.

When $|a| > 1$, the basic sine curve is Stretched.

When $|a| < 1$, the basic sine curve is Shrunk.

The result of this is that the graph of $y = a\sin x$ ranges between $-a$ and a instead of between 1 and -1.

The absolute value of *a* is called the **amplitude** of the function $y = a\sin x$.

The range of the function $y = a\sin x$ for $a > 0$ is $-a \le y \le a$.

Definition of Amplitude of Sine and Cosine Curves

The **amplitude** of $y = a\sin x$ and $y = a\cos x$ represents half the distance between the maximum and minimum values of the function and is given by:

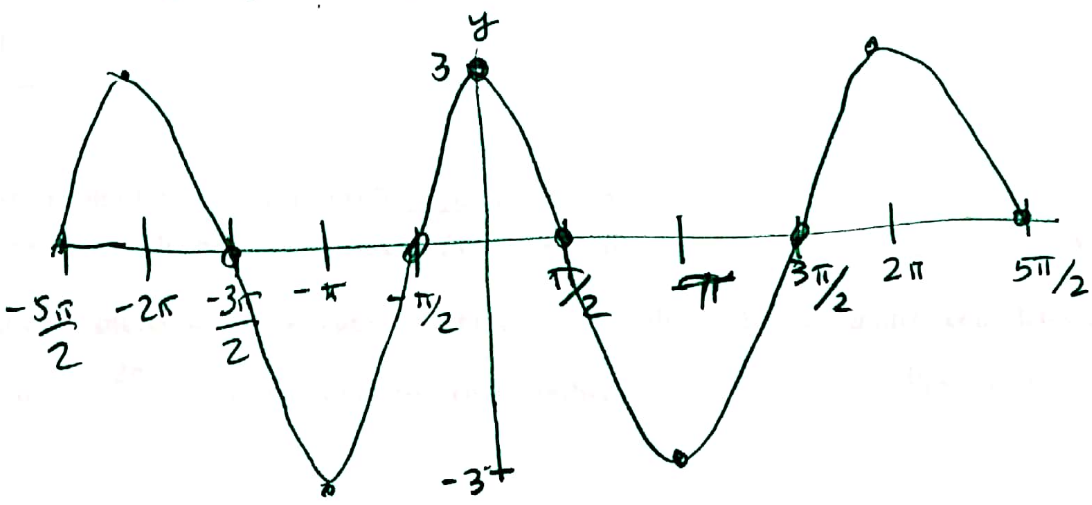
Amplitude= $|a|$. *amplitude can never be negative

When sketching the graphs of the basic sine and cosine functions by hand use the **five key points** in one period of each graph (intercepts, maximum, and minimum).

Axes must be clearly labeled

Ex:1 a.) Sketch the graph of $y = 3\cos x$ on the interval $\left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$.

M - i - m - i - M For 0 to 2π



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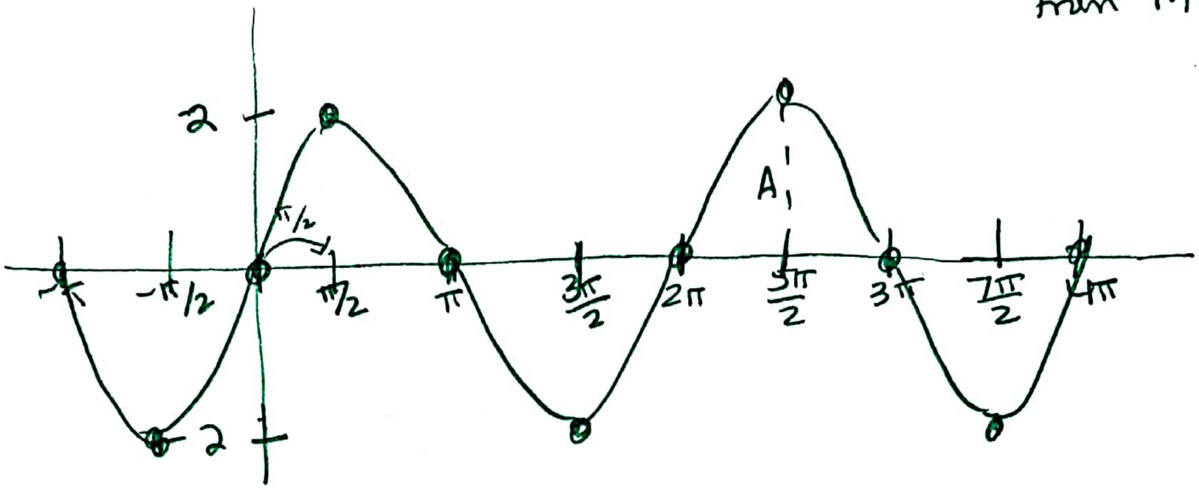
b.) Sketch the graph of $y = 2\sin x$ on the interval $[-\pi, 4\pi]$.

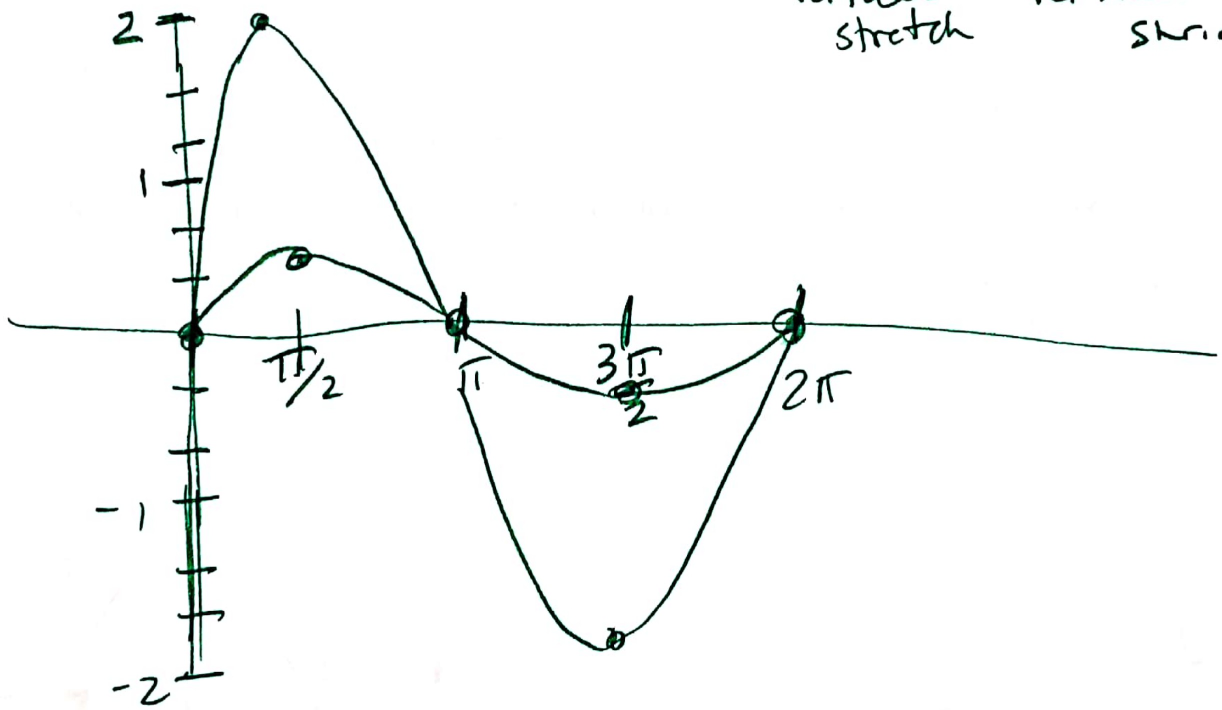
vertical stretch by 2
Amplitude = 2
↑
½ vertical dist from M to m

$y = \sin x$ I - M - I - m - I



Ex:2 On the same coordinate axes, sketch the graphs of $f(x) = 2\sin x$ and $g(x) = \frac{1}{3}\sin x$ for one full cycle of output values $[0, 2\pi]$.

↑ vertical stretch   ↑ vertical shrink



We know that the graph of $y = -f(x)$ is a <u>reflection</u> in the x-axis of the graph of $y = f(x)$. Therefore, the graph of $y = -3\cos x$ is a reflection in the x-axis of the graph of $y = 3\cos x$.

$y = -\cos x$ m I M I m

Because $y = a\sin x$ completes one cycle from $x = 0$ to $x = 2\pi$, it follows that $y = a\sin bx$ completes one cycle from $x = 0$ to $x = \frac{2\pi}{b}$, where b is a positive real number.

$y = -\sin x$ I m I M I

Period of Sine and Cosine Functions

Let b be a positive real number. The **period** of $y = a\sin bx$ **and** $y = a\cos bx$ is given by:

$$\text{Period} = \frac{2\pi}{b}.$$

ALWAYS LABEL BOTH AXES! CRITICAL POINTS MUST BE IDENTIFIED

How many cycles you see from 0 to 2π

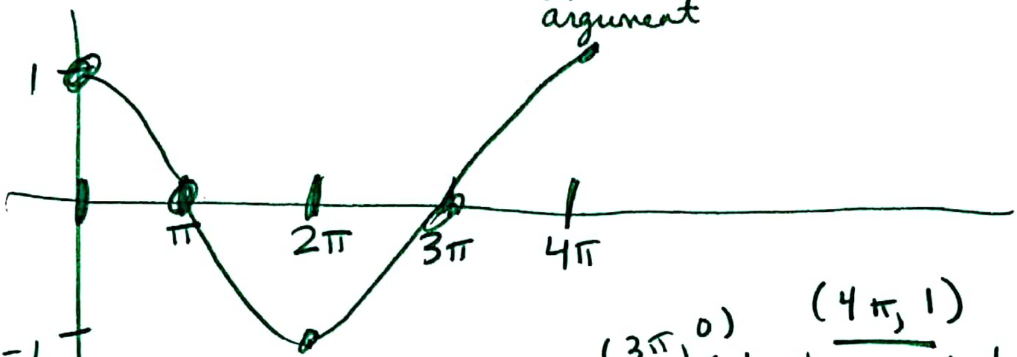
Ex:3 a.) **Sketch the graph of** $y = \cos\frac{x}{2}$ **for one full cycle of output values (one period).**

$y = \cos\underbrace{\left(\frac{1}{2}x\right)}_{\text{argument}}$

$P = \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

one cycle from 0 to 4π

When period changes x values change



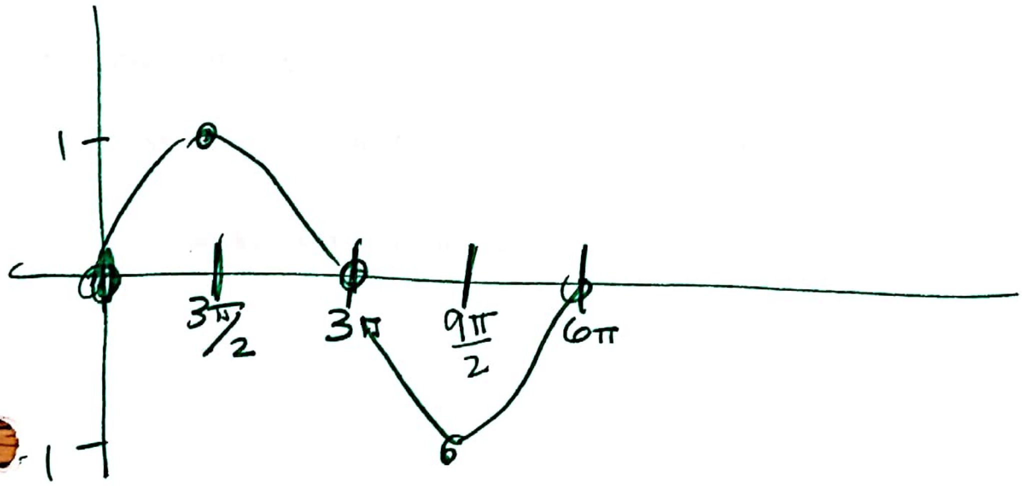
$(4\pi, 1)$

$(0,1)$ $(\pi,0)$ $(2\pi,-1)$ $(3\pi,0)$ list points!

b.) **Sketch the graph of** $y = \sin\frac{x}{3}$ **for one full cycle of output values (one period).**

$y = \sin\frac{1}{3}x$

$P = \dfrac{2\pi}{1/3} = 6\pi$



Must label numerical values on each axis

$(0,0)$ $\left(\frac{3\pi}{2}, 1\right)$ $(3\pi, 0)$ $\left(\frac{9\pi}{2}, -1\right)$ $(6\pi, 0)$

2nd period?